Determination of a flow generating a neutral magnetic mode

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The problem of reconstruction of a flow of conducting incompressible fluid generating a given magnetic mode is considered. We use the magnetic induction equation to derive ordinary differential equations along the magnetic field lines, which give an opportunity to determine the generating flow, if additional data is provided on a two-dimensional manifold transversal to magnetic field lines, and show that an arbitrary solenoidal vector field can not be a neutral magnetic mode sustained by any flow of conducting fluid.

1. Introduction

According to the modern scientific paradigm, magnetic fields of astrophysical objects, ranging from planets to galaxies, are often sustained by conducting fluid flows, driven by convection in the melted medium in their interiors [12, 11, 20, 16, 19]. These processes are governed by the Navier-Stokes and magnetic induction equations (supplemented by other equations, such as heat equation and rheology relations, as appropriate). However, it is difficult to study them numerically because of the extreme parameter values involved, which require prohibitively high resolution of simulations. Thus, application of analytical or semi-analytical methods to the study of astrophysical dynamos appears unavoidable. In the present paper we suggest an approach, in principle enabling one "to separate" the two fundamental equations; hopefully, this can be useful for investigation of asymptotics of astrophysical dynamos.

Usually the magnetic induction equation is employed for investigation of the evolution of magnetic field for a given flow of incompressible conducting fluid (which is predefined in kinematic dynamo problems, or supposed to evolve simultaneously when nonlinear dynamos are studied). We consider here an inverse problem, investigating which consequences existence of a neutral magnetic mode bears upon the generating flow. We show how the flow can be reconstructed uniquely up to the data which must be provided on a two-dimensional manifold transversal to magnetic field lines. We demonstrate that an arbitrary solenoidal vector field can not be a magnetic mode sustained be any flow of incompressible fluid, unless the field satisfies a consistency equation in the fluid volume. We hope that such an analysis may be useful, in particular, for examination of asymptotical properties of various steady magnetohydrodynamic systems and their stability.

In recent numerical studies of nonlinear magnetic dynamos acting in plasma [8] and fluid [13, 5, 6] flows with a prescribed forcing, as well as in thermal convection in a horizontal layer of conducting fluid rotating about a vertical axis [25] or in the absence of rotation [14, 15], it was discovered that temporal evolution can result in emergence of a steady state with a non-vanishing magnetic field. Magnetostatic equilibria in ideal plasma were discussed in [1]. A magnetic field of a steady configuration is a neutral magnetic mode, i.e., a vector field belonging to the kernel of the magnetic induction operator. Neutral magnetic modes play an important rôle in large-scale dynamos [21]-[24].

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We therefore focus on neutral magnetic modes in our analysis. However, a straightforward modification of our approach can be applied for reconstruction of flows for eigenfunctions of the magnetic induction operator associated with any given eigenvalue, or for arbitrary evolving magnetic fields.

2. Reconstruction of flows

Consider the magnetic induction equation

$$\partial_t \mathbf{b} = \eta \nabla^2 \mathbf{b} + \nabla \times (\mathbf{u} \times \mathbf{b}). \tag{1}$$

Magnetic field is solenoidal:

$$\nabla \cdot \mathbf{b} = 0. \tag{2}$$

In a steady state magnetic field is a neutral mode of the magnetic induction operator. For a given flow $\mathbf{b}$ and molecular diffusivity η the operator is elliptic. If magnetic field generation in a bounded volume of fluid is considered and regular boundary conditions for magnetic field are imposed, it has a discrete spectrum, with the eigenvalues tending to $-\infty$. For a randomly chosen pair $\eta, \mathbf{u}$ the kernel of the operator does not contain mean-free magnetic fields, and generically the only mean-free solution is $\mathbf{b} = 0$.

The processes bringing the system to a steady state thus can be viewed as adjustment of the flow to a configuration allowing for a non-zero neutral mean-free magnetic mode. It is natural therefore to treat (1) as an equation in $\mathbf{u}$. "Uncurling" it, one obtains

$$\eta \nabla \times \mathbf{b} = \mathbf{u} \times \mathbf{b} - \eta \nabla a, \tag{3}$$

where a is a scalar function (the constant factor η is introduced for convenience).

Consider separately the components of (3) parallel and perpendicular to $\mathbf{b}$. Scalar multiplying (3) by $\mathbf{b}$ find

$$(\mathbf{b} \cdot \nabla)a = -\mathbf{b} \cdot (\nabla \times \mathbf{b}). \tag{4}$$

The equation controls the magnitude of a magnetic field, whose direction is prescribed: Let $\mathbf{i}_B$ be a unit vector collinear with $\mathbf{b}$, then (4) implies

$$|\mathbf{b}| = -\frac{(\mathbf{i}_B \cdot \nabla)a}{\mathbf{i}_B \cdot (\nabla \times \mathbf{i}_B)}. \tag{5}$$

If a magnetic force line is a closed loop (including the loops emerging due to spatial periodicity), then by virtue of (4)

$$\oint \mathbf{i}_B \cdot (\nabla \times \mathbf{b}) \, ds = -\oint (\mathbf{i}_B \cdot \nabla)a \, ds = 0$$

(the parameter s on the curve is the distance along the curve from a fixed point on it), which can be also viewed as a constraint on the magnitude of the magnetic field (following from (5)).

The component of (3) perpendicular to $\mathbf{b}$ is accessed by cross-multiplication of (3) by $\mathbf{b}$, yielding

$$\eta \, \mathbf{b} \times (\nabla \times \mathbf{b} + \nabla a) = \mathbf{u}|\mathbf{b}|^2 - \mathbf{b}(\mathbf{u} \cdot \mathbf{b}).$$

The component of $\mathbf{u}$ parallel to $\mathbf{b}$ is not determined by (3), hence $(\mathbf{u} \cdot \mathbf{b})$ remains an unidentified arbitrary scalar function. We denote

$$\frac{(\mathbf{u} \cdot \mathbf{b})}{|\mathbf{b}|^2} = 1 + \eta \alpha,$$

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implying
$$\mathbf{u} = (1 + \eta\alpha)\,\mathbf{b} + \eta\,\mathbf{e} \times (\nabla \times \mathbf{b} + \nabla a), \qquad (6)$$

where
$$\mathbf{e} = \mathbf{b}/|\mathbf{b}|^2.$$

(6) and (4) together are equivalent to the equation for a neutral magnetic mode. The scalar field α satisfies the equation

$$(\mathbf{b} \cdot \nabla)\alpha + \nabla \cdot (\mathbf{e} \times (\nabla \times \mathbf{b} + \nabla a)) = 0, \qquad (7)$$

equivalent to the solenoidality condition for the flow $\mathbf{u}$.

Now, in order to find the flow velocity (6), we need to determine ∇a, which we do employing the solenoidality condition for the flow. Generically $\mathbf{i}_B$ and $\nabla \times \mathbf{e}$ are not parallel, and in this case (4) and (7) are equivalent to the equation

$$\nabla a = A\mathbf{i}_A + B\mathbf{i}_B + C\mathbf{i}_C, \qquad (8)$$

where $\mathbf{i}_A, \mathbf{i}_B, \mathbf{i}_C$ is an orthonormal basis,

$$\mathbf{i}_C \equiv \frac{\nabla \times \mathbf{e} - (\mathbf{i}_B \cdot (\nabla \times \mathbf{e}))\,\mathbf{i}_B}{|\nabla \times \mathbf{e} - (\mathbf{i}_B \cdot (\nabla \times \mathbf{e}))\,\mathbf{i}_B|}, \qquad \mathbf{i}_A \equiv \mathbf{i}_B \times \mathbf{i}_C,$$

$$B \equiv -\mathbf{i}_B \cdot (\nabla \times \mathbf{b}) \equiv -|\mathbf{b}|\,\mathbf{i}_B \cdot (\nabla \times \mathbf{i}_B), \qquad (9)$$

$$C = \frac{(B/|\mathbf{b}|)^2 - (\mathbf{b} \cdot \nabla)\alpha - \nabla \cdot (\mathbf{e} \times (\nabla \times \mathbf{b}))}{|\nabla \times \mathbf{e} - (\mathbf{i}_B \cdot (\nabla \times \mathbf{e}))\,\mathbf{i}_B|}. \qquad (10)$$

We derive from (8) individual equations in A and C.

The solvability condition for (8) is obtained by taking its curl:

$$0 = \nabla A \times \mathbf{i}_A + A\nabla \times \mathbf{i}_A + \nabla B \times \mathbf{i}_B + B\nabla \times \mathbf{i}_B + \nabla C \times \mathbf{i}_C + C\nabla \times \mathbf{i}_C. \qquad (11)$$

Scalar multiplying it by $\mathbf{i}_A, \mathbf{i}_B$ and $\mathbf{i}_C$, one finds

$$A = \frac{-C\mathbf{i}_A \cdot (\nabla \times \mathbf{i}_C) - (\mathbf{i}_B \cdot \nabla)C - B\mathbf{i}_A \cdot (\nabla \times \mathbf{i}_B) + (\mathbf{i}_C \cdot \nabla)B}{\mathbf{i}_A \cdot (\nabla \times \mathbf{i}_A)}, \qquad (12)$$

$$0 = A\mathbf{i}_B \cdot (\nabla \times \mathbf{i}_A) + (\mathbf{i}_C \cdot \nabla)A - \frac{B^2}{|b|} + C\mathbf{i}_B \cdot (\nabla \times \mathbf{i}_C) - (\mathbf{i}_A \cdot \nabla)C, \qquad (13)$$

$$C = \frac{-A\mathbf{i}_C \cdot (\nabla \times \mathbf{i}_A) + (\mathbf{i}_B \cdot \nabla)A - B\mathbf{i}_C \cdot (\nabla \times \mathbf{i}_B) - (\mathbf{i}_A \cdot \nabla)B}{\mathbf{i}_C \cdot (\nabla \times \mathbf{i}_C)}, \qquad (14)$$

where B is defined by (9). Substitution of (12) into (14) yields a second order differential equation along magnetic force lines, in principle, defining C. Initial conditions for this equations must be set on two-dimensional manifolds, transversal to magnetic force lines. They must assure geometric consistency: the solutions along closed force lines must be periodic. For force lines, intersecting with the boundary of the region occupied by the fluid, it is naturally to set the conditions on the boundary. The data can be provided on two manifolds, crossing a force line; in this case one obtains a boundary value problem for C. In turn, α can be found, in principle, from (10). This completes reconstruction of the flow. (The divergence of (8) yields an equation in a, which can be used to find

the potential itself.) Substituting A (12) and C into (13), one obtains an equation in $\mathbf{b}$. Thus, not every solenoidal field can be a magnetic neutral mode: The scalar consistency equation (13) constrains, together with the solenoidality condition, a neutral mode up to a scalar field.

Implementation of this program can become particularly difficult in the presence of magnetic nulls, i.e., points, where magnetic field vanishes. (This is clear, of course, already from the definition of the vector field $\mathbf{e}$, which becomes singular at the nulls). Topology of magnetic field with null points and its bifurcations during reconnections are studied in detail in solar magnetohydrodynamics [9, 10, 2, 17, 18] – they are presumed to be of fundamental importance for occurrence of sudden explosive energy release events, solar flares, in the Sun's corona. In the vicinity of a null point magnetic field exhibits an approximately linear behavior controlled by the Jacobian $\|\partial b_i/\partial x_j\|$. Solenoidality of the magnetic field implies, that the sum of the three eigenvalues of this matrix vanishes. Hence, generically it has two eigenvalues with real parts of the same sign, and an eigenvalue of an opposite sign. Consequently, one can identify a two-dimensional manifold of magnetic force lines behaving coherently – all approaching the null point or all departing from it (if the two eigenvalues have negative or positive real parts, respectively) and an one-dimensional manifold (a force line), exhibiting the behavior of the opposite kind. In the parlance of solar physics, the two-dimensional manifold is the *fan*, and the one-dimensional manifold the *spine* of the null (see Fig. 1 in [4]). Therefore, in our problem there are infinitely many characteristics (constituting the fan), which must bring the same values of A and C to (or take the same values from) the null point, implying that the problem of consistency of the global solution for the flow arises. The situation is further complicated by the fact that $\mathbf{i}_B$ is typically discontinuous at null points (its direction is not well-defined), and hence $\mathbf{i}_A$ and $\mathbf{i}_C$ are discontinuous as well.

Thus, the presence of magnetic null points is likely to result in a discontinuity of the reconstructed flow, but they are not the only source of troubles. More generally, our formalism becomes ill-defined at the points, where the magnetic field $\mathbf{b}$ is parallel to $\nabla \times \mathbf{e}$. If a magnetic force line crosses the boundary at two points, a problem arises in satisfying the boundary conditions for the flow at the two points.

3. Axisymmetric magnetic neutral modes

Equations (12)-(14) suggest that the complexity of the problem depends considerably on the geometry of magnetic force lines. For instance, reconstruction of the flow is difficult, if force lines exhibit a chaotic spatial behavior. We consider here one of the simplest examples of an axisymmetric magnetic neutral mode

$$\mathbf{b} = b(\rho, z)\mathbf{i}_\varphi, \qquad \mathbf{i}_B = \mathbf{i}_\varphi,$$

(ρ, φ, z) being a cylindrical coordinate system and $\mathbf{i}_\rho, \mathbf{i}_\varphi, \mathbf{i}_z$ the respective unit vectors.

Before we formulate the system of equations (12)-(14) in the variables A and C, which we need to solve in order to reconstruct the flow (6), we derive some useful properties of the basis $\mathbf{i}_A$, $\mathbf{i}_B$, $\mathbf{i}_C$. Curls of azimuthal and poloidal vector fields independent of φ are, respectively, poloidal and azimuthal; this implies the orthogonality

$$\mathbf{i}_A \cdot (\nabla \times \mathbf{i}_A) = \mathbf{i}_B \cdot (\nabla \times \mathbf{i}_B) = \mathbf{i}_B \cdot (\nabla \times \mathbf{e}) = \mathbf{i}_C \cdot (\nabla \times \mathbf{i}_C) = 0. \tag{15}$$

By a simple calculation,

$$\mathbf{i}_C = \frac{\nabla \times \mathbf{e}}{|\nabla \times \mathbf{e}|} = h\left(-\frac{\partial \kappa}{\partial z}\mathbf{i}_\rho + \frac{\partial \kappa}{\partial \rho}\mathbf{i}_z\right),$$

where

$$\kappa(\rho, z) \equiv \frac{\rho}{b}, \qquad h(\rho, z) \equiv \frac{1}{|\nabla\kappa|};$$

hence

$$\mathbf{i}_A \equiv \mathbf{i}_B \times \mathbf{i}_C = h\nabla\kappa.$$

Therefore,

$$\mathbf{i}_C \cdot (\nabla \times \mathbf{i}_A) = \mathbf{i}_C \cdot (\nabla h \times \nabla\kappa) = 0,$$

since none of the factors in the triple product has an azimuthal component. By vector algebra identities,

$$\mathbf{i}_A \cdot (\nabla \times \mathbf{i}_C) - \mathbf{i}_C \cdot (\nabla \times \mathbf{i}_A) = -\nabla \cdot (\mathbf{i}_A \times \mathbf{i}_C) = \nabla \cdot \mathbf{i}_\varphi = 0,$$

implying

$$\mathbf{i}_A \cdot (\nabla \times \mathbf{i}_C) = \mathbf{i}_C \cdot (\nabla \times \mathbf{i}_A) = 0. \tag{16}$$

Now, scalar multiplying (11) by $\mathbf{i}_A, \mathbf{i}_B$ and $\mathbf{i}_C$ and employing (15) (in particular, $B = 0$) and (16), one obtains equations

$$0 = \frac{\partial C}{\partial\varphi}, \tag{17}$$

$$0 = A\mathbf{i}_B \cdot (\nabla \times \mathbf{i}_A) + (\mathbf{i}_C \cdot \nabla)A + C\mathbf{i}_B \cdot (\nabla \times \mathbf{i}_C) - (\mathbf{i}_A \cdot \nabla)C, \tag{18}$$

$$0 = \frac{\partial A}{\partial\varphi} \tag{19}$$

(which are now significantly simpler than (12)-(14) in the general case). Equations (17) and (19) are equivalent to

$$C = C(\rho, z), \tag{20}$$

$$A = A(\rho, z). \tag{21}$$

For an axisymmetric magnetic field, (10) takes the form

$$\frac{\partial\alpha}{\partial\varphi} = -|\nabla \times \mathbf{e}|C - \nabla \cdot (\mathbf{e} \times (\nabla \times \mathbf{b})).$$

Consequently, (20) and geometric consistency (2π-periodicity of α in φ) imply that $\alpha = \alpha(\rho, z)$ is an arbitrary function (together with the relations (20), (21) and $B = 0$, this formally confirms a physically obvious fact, that a flow generating an axisymmetric magnetic field is necessarily axisymmetric), and (20) is superceded by

$$C = -\frac{\nabla \cdot (\mathbf{e} \times (\nabla \times \mathbf{b}))}{|\nabla \times \mathbf{e}|}. \tag{22}$$

Now A must be determined from (18). We introduce characteristics $(R(s), Z(s))$ in the (ρ, z) half-plane; they satisfy the ODE's

$$\frac{dR}{ds} = -h(R(s), Z(s))\frac{\partial\kappa}{\partial z}(R(s), Z(s)),$$

$$\frac{dZ}{ds} = h(R(s), Z(s))\frac{\partial\kappa}{\partial\rho}(R(s), Z(s)).$$

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Direct differentiation shows that the characteristics are isolines of the scalar field $b(\rho, z)$. Since along a characteristic

$$\mathbf{i}_B \cdot (\nabla \times \mathbf{i}_A) = \frac{\partial h}{\partial z}\frac{\partial \kappa}{\partial \rho} - \frac{\partial h}{\partial \rho}\frac{\partial \kappa}{\partial z} = \frac{1}{h}\left(\frac{\partial h}{\partial z}\frac{dZ}{ds} + \frac{\partial h}{\partial \rho}\frac{dR}{ds}\right) = \frac{1}{h}\frac{dh}{ds},$$

(18) takes the form

$$\frac{d}{ds}(Ah) = f,$$

where

$$f \equiv h((\mathbf{i}_A \cdot \nabla)C - C\mathbf{i}_B \cdot (\nabla \times \mathbf{i}_C))$$

and C is given by (22). Consequently,

$$A(R(s), Z(s)) = \frac{A(R(0), Z(0))h(R(0), Z(0)) + \int_0^s f(R(s'), Z(s'))\, ds'}{h(R(s), Z(s))}. \tag{23}$$

If a characteristic is a closed orbit of period S, geometric consistency implies that over this orbit

$$\int_0^S f(R(s'), Z(s'))\, ds' = 0. \tag{24}$$

Thus, we have determined ∇a and the flow (6) (to the extent this is permitted by the natural non-uniqueness of solutions to (1) in $\mathbf{u}$).

The well-known Cowling antidynamo theorem states that generation of smooth axisymmetric magnetic fields (including steady ones) of finite total energy is impossible. Two proofs of the theorem (following [7] and [3]) are presented in [11]. The demonstrations rely on the equation of total magnetic energy balance derived for a smooth axisymmetric flow of incompressible fluid, provided the normal component of velocity vanishes on the boundary of the region where the fluid resides. To reconcile our results with the Cowling theorem, we note that the flow that we obtain will not satisfy some of these conditions. It may be singular on the circles, where $b = 0$, or κ has extrema (and then $\mathbf{e}$ or h are singular, respectively). If the volume occupied by the flow is bounded, it cannot be guaranteed that the normal component of the fluid velocity vanishes everywhere on the boundary (or, alternatively, enforcing this condition creates a discontinuity in the flow). Hence, the standard procedure employed to establish the total magnetic energy balance equation will reveal additional sources of magnetic energy, which emerge because the flow is not smooth or the surface integral representing the contribution of the advective term does not vanish; under such circumstances the Cowling theorem is unapplicable.

We have presented the analysis of this section mainly as an illustration of how the proposed formalism might be applied to reconstruct flows for less trivial magnetic field configurations. However, in addition, it provides useful information in regard to the following technical issue: Although we have stated at the end of the previous section that (13) is a constraint for a neutral magnetic mode, we have not yet produced any evidence, that the three equations (12)-(14) are independent. Eqns. (17)-(19), which we have derived considering this particular example, demonstrate that (13) is not a consequence of (12), (14) and solenoidality of magnetic field.

4. Concluding remarks

We have shown in Section 2 that reconstruction of an incompressible flow (6) from the structure of a magnetic mode consists of solution of equations (12) and (14) in A and C, followed by solution of (10) in α. These equations are ordinary differential equations along magnetic force lines; thus, the problem becomes complex, if the force lines exhibit a chaotic behavior. For a solenoidal vector field to be a neutral magnetic mode, it must satisfy the constraint (13).

Substitution of (6) into the momentum equation

$$\nu\nabla^2\mathbf{u} + \mathbf{u}\times(\nabla\times\mathbf{u}) - \mathbf{b}\times(\nabla\times\mathbf{b}) - \nabla p + \mathbf{F} = 0$$

yields an equation in $\mathbf{b}$:

$$\nu\nabla^2((1+\eta\alpha)\,\mathbf{b} + \eta\mathbf{e}\times(\nabla\times\mathbf{b}+\nabla a))$$

$$+\eta((1+\eta\alpha)\,\mathbf{b} + \eta\mathbf{e}\times(\nabla\times\mathbf{b}+\nabla a))\times(\nabla\times(\alpha\mathbf{b}+\mathbf{e}\times(\nabla\times\mathbf{b}+\nabla a)))$$

$$+\eta(\alpha\mathbf{b}+\mathbf{e}\times(\nabla\times\mathbf{b}+\nabla a))\times(\nabla\times\mathbf{b}) - \nabla p + \mathbf{F} = 0 \qquad (25)$$

comprising a closed system of equations together with the solenoidality condition (2). Relation (13) now becomes a constraint on the acceptable fluid forcing $\mathbf{F}$.

Analysis of the dependence of steady or evolving magnetohydrodynamic systems on small viscosity and magnetic diffusivity is a notoriously difficult problem. The structure of (25) may turn out to be advantageous for the study of asymptotics of MHD steady states, when the force $\mathbf{F}$ is of the order of small quantities $\nu \sim \eta$, as it is in nonlinear dynamos with energy equipartition [5, 6]. (The form of the scalar factor in front of $\mathbf{b}$ in (6) has been chosen so that all terms in (25) were in this case of the same order of smallness.)

In Section 3 we have considered an example of the reconstruction problem for axisymmetric neutral magnetic modes. This particular case has proved to be highly degenerate: the denominators in (12) and (14) vanish identically, and the respective components of (11) just testify that ∇a is an axisymmetric vector field. Relation (13) does not constrain further the structure of the magnetic field, but rather defines, by (23), the component A of ∇a. Initial conditions $A(R(0), Z(0))$ for solutions (23) of (18) along characteristics can be chosen on curves in the (ρ, z) half-plane, which are transversal to magnetic force lines. The azimuthal component of the flow velocity, $(1+\eta\alpha)b$, is an arbitrary axisymmetric scalar field (in this case it is controlled neither by the magnetic induction equation, nor, due to independence of φ, by the solenoidality condition). Thus the reconstructed flow is unique up to the data which must be specified on two-dimensional manifold(s) (the scalar field α on the (ρ, z) half-plane) and on one-dimensional curve(s) on this half-plane (the initial conditions $A(R(0), Z(0))$).

The initial data must be smooth so that the resultant field A had no singularities. If the topology of isolines of the magnitude of magnetic field b is non-trivial, the smoothness of the initial data is insufficient; for instance, geometric consistency requires that the integral (24) over any closed magnetic force line vanishes. If the axis of symmetry intersects with the volume occupied by the fluid, axisymmetry gives rise to another problem: regularity of the magnetic field implies $b(0, z) = 0$; consequently, the term $\mathbf{e}\times(\nabla\times\mathbf{b})$ in (6) tends to infinity for $\rho \to 0$. Thus, the flow is non-singular only, if initial conditions for A compensate for this singularity.

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